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Closed string tachyons and their implications for non-supersymmetric strings

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Abstract: Closed string tachyons have long been somewhat mysterious. We note that there is often a regime in the classical moduli space in which one can systematically compute the effective action for such fields. In this regime, the tachyon is light, and cannot be integrated out. Instead, one must consider the combined dynamics of gravitons, moduli, tachyons and other light fields. We compute the action and find that the quartic term for the tachyon is positive in the field definition where the tachyon has no derivative coupling to the radion. We study the evolution of isotropic, homogeneous configurations and find that typically the system is driven to regions where the calculation is no longer under control.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua, Tachyon Condensation.

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Contents

1. Introduction

Most known non-supersymmetric string theories contain tachyons. For years there seemed to be no reliable technique in string theory to compute the potential for tachyonic fields (nor was it clear that such a potential is meaningful), and so it was hard to tell whether they are similar to familiar sorts of field theory instabilities, or represent some new phenomena. If the latter, one could imagine that they might even signal some difficulty or inconsistency in these vacua, which would be of great interest.

Luckily, for tachyons in open string theory, the questions above have been largely resolved recently. In particular, the decay of a D-brane anti D-brane pair in the superstring [1], which contains open string tachyons when the brane antibrane separation is smaller than a critical value [2], has been interpreted as tachyon condensation. A similar interpretation applies to the decay of an unstable D-brane of the bosonic string [4, 3]. The off-shell open string tachyon potential can be calculated using the techniques of open string field theory [5], and it contains a local minimum corresponding to the state where D-branes have decayed [3].

The fate of tachyons of closed string theory remains largely mysterious, however. In particular, it is not clear whether there exists a minimum of the tachyon potential similar to the one found in the open string theory. The difficulty in computing the tachyon potential is well known. The potential is a zero-momentum quantity, while the tachyon vertex operator is only defined on shell, far from zero momentum. Closed string field theory is not known well-enough to carry out the necessary off-shell calculations.

There is, however, one set of circumstances in which on can compute the tachyon potential on shell. Many non-supersymmetric theories have, in their classical moduli spaces, critical points where tachyons appear/disappear. The simplest such case to study is that due to Rohm [6]; more extensive examples have been enumerated in [7]. In Rohm's model, type-II string theory is compactified on a circle (torus), with boundary conditions such

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that bosons are even but fermions odd in the extra dimension (Scherk-Schwarz boundary conditions). At large radius, there is no tachyon. As the radius decreases, however, a tachyon appears at a critical radius, R_o. At the critical radius, the "tachyon to be" is a massless particle, and it is straightforward to compute its effective action.

The analogous situation in the open superstring arises when a D-brane and an anti D-brane are at a critical separation such that an open string connecting them is massless. The effective potential for such modes was calculated in [8] with the conclusion that the quartic term is positive, in agreement with the idea that these modes start to condense when the D-brane separation is reduced below the critical one.

In this paper we carry out an analogous calculation for the Rohm compactification of the closed string at the critical radius, and find results similar to those found for the open string. At leading order in α', there is a quartic coupling of the tachyon, as well as a higher derivative coupling to the radion. With suitable field redefinitions, the quartic term is positive. If Δ measures the distance to the critical point (in the tachyonic direction), for small Δ these results remains correct to order Δ.

Since, for small Δ, the tachyon potential has a local minimum, it is tempting to then integrate out the tachyon, and obtain a potential, already at the classical level, for the moduli. However, as we will see, this is not consistent. Near the critical point, the moduli and the tachyon must be retained as light fields.[1] Indeed, this is why the value of the quartic coupling is convention-dependent. Field redefinitions can render the coupling negative, while introducing various derivative couplings to moduli.

In light of these comments, we focus on the evolution of homogeneous field configurations of tachyons, moduli and gravitons near the critical point. This evolution is independent of possible field redefinitions. Not surprisingly, we find that the system is quickly driven out of the regime where the effective action is calculable. One is driven to values of the radii where the tachyon is no longer nearly marginal; in addition, one is driven to a regime where the theory is strongly coupled and the metric is becoming singular.

The classical moduli, beyond the critical point, are not moduli at all. As one moves out of the region of controlled approximation, the curvature of their potential is of order string scale. In this interior region of the "moduli space," in other words, not only is the system strongly coupled, but it is not clear what degrees of freedom should be used to describe it.

Still, recent developments in string duality suggest some plausible speculations. Strong-weak coupling duality of supersymmetric string theories suggests that in the very strongly coupled region, the system becomes weakly coupled, and the potential again tends to zero (from below). So there is likely some sort of minimum. The AdS/CFT correspondence suggests that this minimum is described by an isolated conformal field theory.[2] One has no particular reason to think that this system is supersymmetric, but one is free to speculate on this possibility.

[1] A similar conclusion was reached in studies of effective actions for Melvin backgrounds, which are generalizations of the Rohm compactifications [9]. It was found, after a T-duality transformation, the tachyon fits with other fields into a low-energy effective action.

[2] For related ideas on the end-point of closed string tachyon condensation, see [10].

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What this analysis suggests is that the theory has some number of stable, possibly non-supersymmetric, AdS states in its interior. Actually, AdS may, in some sense, be a misnomer, since there is no parameter which makes a semiclassical, general relativistic analysis valid. Perhaps one should refer to these systems simply as conformal field theories, in some strongly coupled domain.

2. Rohm's compactification

Rohm proposed a simple model of supersymmetry breaking in string theory [6]. Take type-II theory compactified on a circle, and impose antiperiodic (Scherk-Schwarz) boundary conditions on fermions. This breaks supersymmetry. It leads, at one loop, to a potential for the moduli. But for our purposes, what is more interesting is that, while for large radius there is no tachyon, a tachyon does appear at a finite value of the radius. In the RNS formulation, for example, the GSO projection is different in sectors with even and odd winding. Alternatively, in the Green-Schwarz formulation, the Green-Schwarz fermions are antiperiodic in the odd winding number sectors. In general, for bosons, the momenta are ($\alpha' = 1/2$)

$$k_L = \frac{m}{2R} - nR\,, \qquad k_R = \frac{m}{2R} + nR\,. \tag{2.1}$$

Thus, for example, in the sector with winding number one, the mass formula is ($\alpha' = \frac{1}{2}$):

$$L_0 = k_L^2 - 1 = n^2R^2 - 1 \tag{2.2}$$

where in the last expression we have taken $m = 0$. So at $R = 1$, a tachyon appears. At smaller radius, more and more tachyonic modes appear. The small radius limit of the theory is in fact T-dual to a ten-dimensional theory with a tachyon known as the type 0 string theory [11].

There may exist another interpretation of our results where the compact dimension is the euclidean time. This interpretation applies to the thermal string theory, and the critical radius corresponds to the Hagedorn phase transition [12]–[15]. Formally, the calculations which follow can be applied to the system near the Hagedorn transition. The interpretation of these results, however, raises a number of subtle issues, which we leave for future work.[3]

3. The effective action

At the critical point, the would-be tachyon is massless, so we can calculate its effective action using on-shell vertex operators. In the RNS formulation, for example, the tachyon vertex operator in the minus one ghost number picture is particularly simple [16]:

$$V_{-1} = e^{2ik_LX_L+2ik_RX_R}e^{ip\cdot x}e^{-\gamma} \tag{3.1}$$

whereas in the ghost number zero picture it is

$$V_0 = (p\cdot\psi - 2k_L\psi_5)(p\cdot\tilde{\psi} - 2k_R\tilde{\psi}_5)e^{2ik_LX_L+2ik_RX_R}e^{ip\cdot x}\,. \tag{3.2}$$

[3]Willy Fischler and Joe Polchinski in unpublished work also noted some of these issues. We thank Joe Polchinski for discussions.

Take the initial momenta, (k_L, k_R, p) to be $(1,-1,p_1),(-1,1,p_2)$ and the final momenta to be $(-1,1,p_3),(1,-1,p_4)$ (all momenta are defined to flow into the diagram). Then the scattering amplitude is obtained from

$$\begin{aligned}
&\langle V_{-1}(k_1,p_1,z_1)V_{-1}(k_2,p_2,z_2)V_0(k_3,p_3,z_3)V_0(k_4,p_4,z_4)\rangle = \\
&\quad = \frac{|z_1-z_4|^2}{|z_1-z_2|^4}\frac{|z_2-z_3|^2}{|z_2-z_4|^2|z_3-z_4|^4|z_1-z_3|^2}\Pi|z_i-z_j|^{-\frac{p_i\cdot p_j}{2}}\left(-4+\frac{s}{2}\right)^2
\end{aligned} \tag{3.3}$$

where the last factor comes from the contraction of the fermions, and s,t,u are the Mandelstam variables:

$$s=-(-p_1+p_2)^2\,,\qquad t=-(p_1+p_3)^2\,,\qquad u=-(p_1+p_4)^2\,.$$

Taking $z_1=\infty, z_2=0, z_3=1, z_4=z$, the amplitude is

$$\begin{aligned}
\mathcal{A} &= c\int d^2z\frac{(1+s/8)^2}{|1-z|^{4+s/4}|z|^{2+t/4}} \\
&= c'\frac{\Gamma(-t/8)\Gamma(-s/8)(1-u/8)\Gamma(1-u/8)}{\Gamma(-1+u/8)\Gamma(1+t/8)\Gamma(1+s/8)} \\
&= c'\frac{u}{ts}\left(1-\frac{u}{8}\right)^2\frac{\Gamma(1-t/8)\Gamma(1-s/8)\Gamma(1-u/8)}{\Gamma(1+u/8)\Gamma(1+t/8)\Gamma(1+s/8)} \\
&= c'\left[\frac{1}{t}+\frac{1}{s}+\left(\frac{s}{4t}+\frac{t}{4s}\right)+\frac{1}{2}+\frac{s^2}{64t}+\frac{t^2}{64s}+\frac{3s}{64}+\frac{3t}{64}+\cdots\right].
\end{aligned} \tag{3.4}$$

In the last step, we have used energy-momentum conservation, $u=-t-s$; the terms we have dropped are suppressed by two powers of s,t.

In order to obtain the contact terms in the effective action, we need to subtract the graviton, gauge boson and radion exchanges. The graviton exchange (which we will use to normalize the string amplitude) can be obtained from the effective action, following closely the analysis of [16]. Expanding the gravitational action, we find

$$\int d^9x\frac{1}{2}[h_{mn}(\Delta+\cdots)h_{mn}+\frac{1}{2}\phi\Delta\phi^*-\mathcal{T}_{mn}h_{mn}] \tag{3.5}$$

where

$$\Delta=-\partial^2\,,\qquad g_{mn}=\delta_{mn}+h_{mn},$$

and

$$\mathcal{T}_{mn}=\frac{1}{2}\left[\partial_m\phi^*\partial_n\phi+\partial_n\phi^*\partial_m\phi-\delta_{mn}(\partial_k\phi^*)(\partial^k\phi)\right]. \tag{3.6}$$

Integrating out the graviton to leading order, we get the exchange amplitude which may be written as a contribution to the S-matrix generating functional $\mathcal{S}(\phi)$

$$\begin{aligned}
\mathcal{S}(\phi) &= \int d^9x\left[\frac{1}{2}\phi\Delta\phi^*+W\right], \\
W &= -\frac{1}{2}\mathcal{T}_{mn}\Delta^{-1}_{mn,kl}\mathcal{T}_{kl}\,,
\end{aligned}$$

where $\Delta^{-1}_{mn,kl} = (\delta_{m(k}\delta_{l)n} - \frac{1}{D-2}\delta_{mn}\delta_{kl})\Delta^{-1}$ is the graviton propagator in the harmonic gauge. Evaluating the contractions, we find

$$\begin{aligned} W &= \int \Pi \frac{d^9 k_i}{(2\pi)^9} e^{ik_i\cdot x} \mathcal{W}(k_1,k_2,k_3,k_4)\phi(k_1)\phi(k_2)\phi^*(k_3)\phi^*(k_4)\,, \\ \mathcal{W} &= \frac{1}{32}\left[\frac{s^2+u^2}{t} + \frac{u^2+t^2}{s} - (t+s)\right]. \end{aligned} \tag{3.7}$$

The non-derivative radion coupling to the tachyon can be determined from the mass formula for the tachyon: $m^2 = 4R^2 - 1$. There also may be derivative couplings which are not visible in the on-shell 3-point function for ϕ, ϕ^* and the radion. We can obtain both couplings by examining the OPE of two tachyons directly. This includes:

$$V_0(k_L,k_R,p)V_0(-k_L,-k_R,p') \sim \frac{4k_Lk_R}{|z-z'|^2}\left[k_L^2 + \frac{p\cdot p'}{4}\right]\left[k_R^2 + \frac{p\cdot p'}{4}\right]\partial X_L \bar{\partial} X_R e^{i(p+p')\cdot x} + \cdots . \tag{3.8}$$

To get the gauge boson couplings, replace $2k_L\partial x_L$ by $p_\mu \partial x_L^\mu$, and similarly for the right.

So radion exchange gives a contribution,

$$\frac{1}{2s}\left(1+\frac{s}{8}\right)^4 + \frac{1}{2t}\left(1+\frac{t}{8}\right)^4 . \tag{3.9}$$

Gauge boson exchanges give a contribution

$$\frac{t+s/2}{s} + \frac{s+t/2}{t} .$$

We can make these contributions manifest by rewriting the scattering amplitude as

$$\begin{aligned} &c'\left[\left(\frac{1}{t}+\frac{1}{2}\right) + \left(\frac{1}{s}+\frac{1}{2}\right) + \left(\frac{s}{4t}+\frac{1}{8}\right) + \left(\frac{t}{4s}+\frac{1}{8}\right) - \right. \\ &\qquad \left. -\frac{1}{4} + \left(\frac{(s+u)^2}{32t} + \frac{(t+u)^2}{32s} - \frac{(t+s)}{32} + \frac{3s+3t}{128}\right)\right] = \\ &\qquad\qquad = c'\left[\text{radion} + \text{gauge boson} - \frac{3}{4} + \frac{3s+3t}{128}\right]. \end{aligned} \tag{3.10}$$

The $-3/4$ represents a non-derivative quartic interaction; the last term is a quartic interaction with two derivatives. To determine the sign of the quartic term, one just has to compare with the expected sign from, e.g. the radion exchange. A positive quartic coupling should give a contribution of the same sign as the pole term; so we have found a negative quartic coupling.

4. Field redefinitions

The effective action is only defined up to field redefinitions. The operator product expansion of eq. (3.8) suggests that one take the coupling

$$\partial^2 R|\phi|^2 + \frac{3}{8}|\phi|^4 + \text{four derivative terms}. \tag{4.1}$$

In this case, the quartic coupling is negative. Alternatively, one can make a field redefinition,

$$R \to R - \frac{1}{2}|\phi|^2 \,. \tag{4.2}$$

This yields

$$\mathcal{L}_{\rm eff} = \frac{1}{16}(\phi^*\phi)^2 + \frac{1}{64}\phi^*\phi\partial_\mu\phi^*\partial^\mu\phi \,. \tag{4.3}$$

This effective action, along with radion, gauge boson, and graviton exchanges reproduces the amplitude of eq. (3.4). In this form, the quartic coupling is positive.

This form of the action could be derived directly had we taken the radion exchange contribution of the form

$$\frac{1}{2s} + \frac{1}{2t} \tag{4.4}$$

rather than (3.9). Then the amplitude with all massless exchanges subtracted is

$$\frac{1}{4} + \frac{s+t}{32} \tag{4.5}$$

corresponding to $\mathcal{L}_{\rm eff}$.

It is clear from this discussion that the sign of the quartic coupling is ambiguous. However, the low energy dynamics is not. We will discuss these issues in section 6.

5. Another model

There are other models to which one can apply this sort of analysis. One example is provided by the Rohm compactification, not for the type-II theory, but for the heterotic theory (for definiteness, we can discuss the O(32) theory; it is convenient to use the fermionic formulation for the left movers).

The analysis is quite similar to the type-II case. There is again a tachyon at the (same) critical radius, in the vector representation of SO(32). In the ghost number -1 picture, we can take the vertex operator to be

$$V_{-1} = \lambda_a e^{ik_L X_L + ik_R X_R + ip\cdot x} e^{-\gamma} \,. \tag{5.1}$$

We can now make the computation of the four point function very similar to that of the type-II case, if we judiciously choose the quantum numbers of the tachyons. One can take, say, the two operators in the -1 picture to carry gauge index 1, and the two in the zero picture to carry gauge index 2. Then the calculation of the amplitude is identical to that in the type-II case, except that the factor $(1+s/8)^2$ is now $(1+s/8)$. In extracting the quartic coupling, however, we need to note that the massless exchanges are different. First, the couplings of the radial mode are now proportional to $(1+s/8)$, etc. The quantum numbers of the exchanged particles are also different. In the t-channel, for example, the exchanged particle carries $O(32)$ quantum numbers; it is a dimensionally reduced gauge boson. Similarly, the quantum numbers of the exchanged gauge bosons in the two channels are different. One again obtains derivative couplings of the radion to the tachyon, and a negative quartic coupling. A suitable field redefinition eliminates the derivative couplings, and yields a positive quartic coupling.

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6. Tachyon and moduli dynamics; speculations on the "moduli potential"

We have seen in two examples that the question of whether the tachyon potential has a calculable minimum is not unambiguous. Field redefinitions allow us to absorb part of the coefficient of the quartic coupling into operators such as $\partial^2 R|\phi|^2$. The low energy physics should be invariant under these redefinitions. If the quartic potential were positive, one might have hoped to find the minimum of the tachyon potential and integrate it out. This would leave the puzzle of field redefinitions. In fact, integrating out the tachyon in this way, at least in the regime where we know how to calculate the tachyon effective action, is not consistent.

To see this, suppose that the tachyon potential has a minimum. If for the moment we call $R^2 = 1 - \sigma$, then minimizing with respect to the tachyon, the potential behaves as

$$V = -\frac{\sigma^2}{g^2}\,. \tag{6.1}$$

This is a potential for the radion and the dilaton. But the effective mass for the σ field is of order one. Before proceeding further, one observation about this potential is in order. It might seem that this potential becomes arbitrarily negative for weak coupling. But there is no simple meaning to energy in general relativity, and to interpret this result it is helpful to go to the Einstein frame. There, in any dimension, the potential depends on g with a positive power. So if there is a minimum, or even a regime where the potential is negative, the system is driven to strong coupling, where any analysis is likely to break down.

In any case, given our result that the tachyon cannot consistently be integrated out near the critical point, it is necessary to keep both the fields σ and ϕ (as well as the dilaton) in the low energy effective action in this region of field space. We have solved for the evolution of homogeneous and isotropic field configurations with the equations following from this action. These equations are manifestly covariant under field redefinitions. We have studied a variety of possible initial conditions. For example, we have taken the initial radius such that the tachyon has a small negative mass-squared, and started the tachyon near the minimum of the effective potential with the positive quartic coupling. Not surprisingly, the system is driven, with typical initial conditions, to a regime in which the tachyon is no longer approximately marginal (and in which there are, in fact, more tachyons) and in which the coupling is becoming strong.

To summarize: at this stage, one has generated, *classically*, a potential on the moduli space. In other words, the moduli are not moduli past the critical point. It is necessary, near the critical point, to study the dynamics of the original moduli and tachyon together. One is quickly driven, however, to a regime where one does not have control of the calculation.

We can speculate on the form of the moduli potential in the strongly coupled region. If we suppose that there is an S-duality, then we might expect that there is a minimum in this region. It might be described by some suitable conformal field theory. There is no obvious small parameter, so this minimum will occur when all scales are comparable and all couplings of order one.

The alternative is that the potential is unbounded below. This would be an exciting possibility, and would establish that these vacua are not sensible, but it seems unlikely. It

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is true that at the minimum, if it exists, the appropriate degrees of freedom to describe the system are not the radion and dilaton, but others. An isolated CFT is probably a good model for this.

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